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Computational Analysis of Magnetohydrodynamic Flow Behaviors and Heat Transfer Characteristics of Dissipative Casson-Carreau Nanofluid over a Stretching Sheet with Internal Heat Generation

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Abstract: The present paper focuses on the application of Chebyshev pseudospectral collocation method to the study of simultaneous effects thermal radiation, inclined magnetic field and temperature-dependent internal heat generation on unsteady two-dimensional flow and heat transfer analysis of dissipative Casson-Carreau nanofluid over a stretching sheet embedded in a porous medium. With the aid of similarity variables, the systems of governing partial differential equations are reduced to system of nonlinear third and second orders ordinary differential equations which are solved numerically by the collocation method. Additionally, effects of the thermal-fluidic parameters on the flow behaviors and the heat transfer characteristics of the nanofluids are investigated and presented. The present work will help in manufacturing industries and production engineering.

Keywords: MHD, Nanofluid, non-uniform heat source/sink, Carreau fluid; thermal radiation and free convection; Chebyshev pseudospectral collocation method

1. Introduction

The application of fluid flow over a stretching surface has been well established in polymer devolatilization and processing, wire and fiber coating, heat exchangers, extrusion process, polymer extrusion, wire drawing, metal and plastic extrusion, continuous casting, glass fiber production, crystal growing and paper production, chemical processing equipment, etc. The flow of fluid over such surfaces that are susceptible to stretching has been studied analytically [1-11]. However, some other researchers have put forward numerical studies to explore the flow phenomena [12-21]. The significance of thermal radiation and variable viscosity on natural convection over a stretching surface has been examined by some [22-35]. Other researchers went on to present their views on the interesting topic by adapting nanofluid into the fluid process [36-54]. In recent times, due to the wide and important areas of applications, the flow behaviors of Carreau fluid have continued to arouse research interests [55-66].

The review studies have been analyzed using approximate analytical, semi-analytical and numerical methods. Among the numerical methods, the numerical solutions using Pseudospectral collocation method represent efficient ways of obtaining solutions to nonlinear problems even with complexities in the boundary conditions [67-78]. Therefore, Chebyshev pseudospectral collocation method is applied to the study of simultaneous effects thermal radiation, inclined magnetic field and temperature-dependent internal heat generation on unsteady two-dimensional flow and heat transfer analysis of dissipative Casson-Carreau nanofluid over a stretching sheet embedded in a porous medium. With the aid of similarity variables, the systems of governing partial differential equations are reduced to a system of nonlinear third and second orders ordinary differential equations which are solved numerically by the collocation method. The results of the numerical method are verified with the results of convectional numerical method using Runge-Kutta fourth-order and shooting method. Parametric studies of the flow behaviors and the heat transfer characteristics of the nanofluids are presented.

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2. Problem Formulation

From the transient, two-dimensional boundary layer fluid flow of a conducting and heat generating Casson and Carreau nanofluids over a sheet susceptible to stretching extreme by liquid film of uniform size (thickness) *h(t)* as represented schematically in Fig.1. The stretching velocity along x-axis is *U(x,t)* and y–axis is perpendicular to it with dissipation and volume fraction considered.

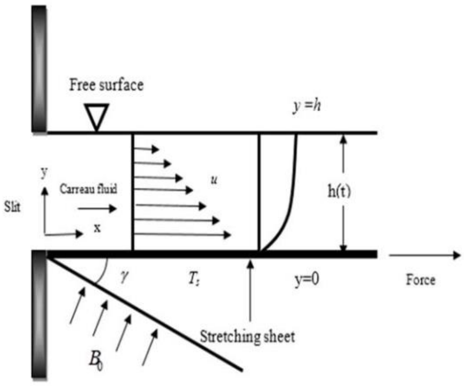

Fig. 1 – Flow geometry of the problem [40]

The equations for continuity and motion are,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu_{nf}\left(1+\frac{3(n-1)\Gamma^2}{2}\left(\frac{\partial u}{\partial y}\right)^2\right)\frac{\partial^2 u}{\partial y^2}-\sigma B_o^2 u cos^2\gamma \tag{2}$$

$$\left(\rho C_p\right)_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{nf}\frac{\partial^2 T}{\partial y^2}+\mu_{nf}\left(\frac{\partial u}{\partial y}\right)^2+q''' \tag{3}$$

where,

$$\rho_{nf}=\rho_f(1-\phi)+\rho_s\phi$$

$$\left(\rho C_p\right)_{nf}=\left(\rho C_p\right)_f(1-\phi)+\left(\rho C_p\right)\phi$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}$$

$$k_{nf}=k_f\left[\frac{k_s+2k_f-2\phi\left(k_f-k_s\right)}{k_s+2k_f+\phi\left(k_f-k_s\right)}\right]$$

Assuming no slip condition, the appropriate boundary conditions are given as,

$$u=U_w,\quad v=0,\quad T=T_s \quad \text{at} \quad y=0$$

$$\frac{\partial u}{\partial y}=0,\quad \frac{\partial T}{\partial y}=0,\quad y=h$$

where,

$$v=\frac{dh}{dt}=-\frac{\alpha\beta}{2}\left(\frac{\nu_f}{b(1-\alpha t)}\right)^{\frac{1}{2}}$$

$$y=h(t)=-\int\left\{\frac{\alpha\beta}{2}\left(\frac{\nu_f}{b(1-\alpha t)}\right)^{\frac{1}{2}}\right\}dt$$

It should be stated at this juncture that the mathematical problem is implicitly formulated only for $x \geq 0$. In other to avoid the complications due to surface waves, a further assumption is made that the surface of the planar liquid film is smooth. Also, the influence of interfacial shear due to the quiescent atmosphere i.e., the effect of surface tension is assumed to be negligible. The viscous shear stress $\tau=\mu\left(\partial u/\partial y\right)$ and the heat flux $\dot{q}''=-k\left(\partial T/\partial y\right)$ vanish at the adiabatic free surface (*at y = h*). The boundary conditions used in this present work are in line with the previous works of Kumar et al. [40] and Abel et. al. [53].

The non-uniform heat generation/absorption, q''' is taken as,

$$q'''=\frac{k_f U_w}{x\nu_f}\left[A^*\left(T_s-T_o\right)f'+B^*\left(T_s-T_o\right)\right] \tag{4}$$

Where the surface temperature T_s of the stretching sheet varies with respect to distance *x*-from the slit as,

$$T_s=T_o-T_{ref}\left(\frac{bx^2}{2\nu_f(1-at)^{\frac{3}{2}}}\right) \tag{5}$$

and the stretching velocity varies with respect to *x* as,

$$U=\frac{bx}{(1-at)} \tag{6}$$

On introducing the following stream functions

$$u=\frac{\partial\psi}{\partial y},\quad v=\frac{\partial\psi}{\partial x}$$

And the similarity variables,

$$u=\frac{bx}{(1-at)}f'(\eta,t),\quad v=-\left(b\nu_f\right)^{-\frac{1}{2}}(1-at)^{-\frac{1}{2}}f(\eta,t)$$

$$\eta=\left(b/\nu_f\right)^{\frac{1}{2}}(1-at)^{-\frac{1}{2}}y$$

$$T=T_o-T_{ref}\left(bx^2/2\nu_f\right)(1-at)^{-\frac{3}{2}}\theta(\eta)$$

Substituting the stream function and the similarity variables into Eqns. (1), (2) and (3), we have a partially coupled third and second orders ordinary differential equation.

$$f'''\left\{1+\frac{3(n-1)We\left(f''\right)^2}{2}\right\}+B_1\left\{B_2\left(\begin{array}{c}S\left(f'+\frac{\eta}{2}f''\right)\\+ff''-\left(f'\right)^2\end{array}\right)\right\}-Ha^2 f'cos^2\gamma=0 \tag{7}$$

$$B_3\theta'' + \frac{EcPr}{B_1}\left(f''\right)^2 + \left(A^* f' + B^*\theta\right) - B_4 Pr\left\{\frac{S}{2}\left(\left(\eta\theta' + 3\theta\right) + 2f'\theta - f\theta'\right)\right\} = 0 \tag{8}$$

where,

$$We^2 = \frac{b^3x^2\Gamma^2}{v_f\left(1-at\right)^3},\quad Pr = \frac{\mu c_p}{k_f},\quad Ha^2 = \frac{\sigma B_o^2}{\rho_f b},$$

$$Ec = \frac{U_w^2}{c_p\left(T_s - T_0\right)},\quad S = \frac{\alpha}{b},\quad R = \frac{4\sigma^* T_0^3}{k^* k_f},\quad B_1 = \left(1-\phi\right)^{2.5}$$

$$B_2 = 1-\phi+\phi\frac{\rho_s}{\rho_f},\quad B_3 = \frac{k_{nf}}{k_f},\quad B_4 = 1-\phi+\phi\frac{\left(\rho c_p\right)_s}{\left(\rho c_p\right)_f}$$

And the boundary conditions become,

$$\eta = 0,\quad f = 0,\quad f' = 1,\quad \theta = 0$$

$$\eta = \beta,\quad f = \frac{S\beta}{2},\quad f'' = 0,\quad \theta' = 0$$

3. Solution Method: Chebyshev Pseudospectral Collocation Method

Equations (7) and (8) are systems of coupled non-linear ordinary differential equations which are to be solved alongside boundary conditions. To solve the nonlinear equations, we applied Chebyshev pseudospectral collocation method. Following the method procedure, the η direction domain is approximated by [*0, L*], where *L* is the edge of the boundary layer. Using the algebraic mapping *η=(γ+1) L/2*, the physical region can be mapped out into Chebyshev collocation method computation domain of [*-1, 1*]. The unknown variables in Eqns. (7) and (8) are approximated using the following truncated series of polynomials:

$$f\left(\lambda\right) = \sum_{k=0}^{N}\hat{f}_k T_k\left(\lambda\right),\qquad \theta\left(\gamma\right) = \sum_{k=0}^{N}\hat{\theta}_k T_k\left(\lambda\right) \tag{9}$$

where the values of $\hat{f}_k$ and $\hat{\theta}_k$ are the coefficient of the series to be determined, $\hat{T}_k$ is the Chebyshev polynomial of degree k which is defined on the interval λ ϵ [*-1, 1*].

$$T_k\left(\lambda\right) = cos\left(k cos^{-1}\lambda\right),\quad k = 0,1,...$$

The application of pseudospectral collocation method requires that the spatial discretization of the dimensionless models should be carried out through the utilization of Chebyshev-Gauss-Lobato collocation points. Using the Gauss-Lobato collocation points to define the Chebyshev nodes in [-1,1] as,

$$\lambda_j = cos\frac{\pi j}{N},\quad -1 \le \gamma \le 1,\ j = 0,1,...,N$$

The collocation points are mapped out in computation domain of [*-1, 1*]. To transform any arbitrary (*X, [0, 1]*) interval into standard interval (*ψ*, [*-1, 1*]), a transformation using algebraic mapping X = *(ψ +1) /2* is adopted. The derivatives of the functions *f* and *θ* at the collocation points are represented as,

$$\frac{df}{d\eta} = \frac{2}{L}\sum_{k=0}^{N}\mathbf{D}_{jk}\hat{f}_k \qquad \frac{d\theta}{d\eta} = \frac{2}{L}\sum_{k=0}^{N}\mathbf{D}_{jk}\hat{\theta}_k \tag{10}$$

where **D** is the Chebyshev spectral method differentiation matrix. Higher order derivatives are computed as multiple powers of **D**, i.e.,

$$\frac{d^i f}{d\eta^i} = \left(\frac{2}{L}\right)^i\sum_{k=0}^{N}\mathbf{D}^i_{jk}\hat{f}_k \qquad \frac{d^i\theta}{d\eta^i} = \left(\frac{2}{L}\right)^i\sum_{k=0}^{N}\mathbf{D}^i_{jk}\hat{\theta}_k \tag{11}$$

where *i* is the order of the derivatives.

On substituting Eqns. (9) - (11) into the governing equations in Eqns. (7) and (8) and the boundary conditions respectively, the following equations are obtained,

$$\bar{\mathbf{D}}^3\mathbf{F}\left\{1+\frac{3\left(n-1\right)We\left(\bar{\mathbf{D}}^2\mathbf{F}\right)^2}{2}\right\} + B_1\left\{B_2\left(S\left(\bar{\mathbf{D}}\mathbf{F} + \frac{\eta}{2}\bar{\mathbf{D}}^2\mathbf{F}\right) + \mathbf{F}\bar{\mathbf{D}}^2\mathbf{F} - \left(\bar{\mathbf{D}}\mathbf{F}\right)^2\right)\right\} - Ha^2\left(\bar{\mathbf{D}}\mathbf{F}\right)cos^2\gamma = 0 \tag{12}$$

$$B_3\bar{\mathbf{D}}^2\Theta + \frac{EcPr}{B_1}\left(\bar{\mathbf{D}}^2\mathbf{F}\right)^2 + \left(A^*\bar{\mathbf{D}}\mathbf{F} + B^*\Theta\right) - B_4 Pr\left\{\frac{S}{2}\left(\left(\eta\bar{\mathbf{D}}\Theta + 3\Theta\right) + 2\Theta\bar{\mathbf{D}}\mathbf{F} - \mathbf{F}\bar{\mathbf{D}}\Theta\right)\right\} = 0 \tag{13}$$

where,

$$\bar{\mathbf{D}} = \left(2/L\right)\mathbf{D},\ \mathbf{F} = \left\{\hat{f}_0, \hat{f}_1, ..., \hat{f}_N\right\}^T,\ \Theta = \left\{\hat{\theta}_0, \hat{\theta}_1, ..., \hat{\theta}_N\right\}^T$$

and the superscript T denotes transpose. The boundary conditions become,

$$\hat{f}_N = 0,\quad \sum_{k=0}^{N}\hat{\mathbf{D}}_{Nk}\hat{f}_k = 1,\quad \hat{\theta}_N = 0$$

$$\sum_{k=0}^{N}\hat{\mathbf{D}}_{0k}\hat{f}_k = \frac{S\beta}{2},\quad \sum_{k=0}^{N}\hat{\mathbf{D}}^2_{Nk}\hat{f}_k = 0,\quad \sum_{k=0}^{N}\hat{\mathbf{D}}_{Nk}\Theta_k = 0$$

Although, the algorithm for the implementation of the Chebyshev pseudospectral collocation method is given in the next section, it should be stated that Eqn. (12) can be solved separately subject to the given boundary conditions using a MATLAB nonlinear equation solver *fzero* which is based on the quasi-Newton method. Also, it should be noted that Eqn. (7) or Eqn. (12) is a third order differential equation. There are four boundary conditions for this equation. One of the ways to make the governing equation to satisfy all the boundary conditions is to differentiate the governing equation in Eqn. (7) or Eqn. (12) to fourth-order.

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Once, the solution for **F** is obtained, it is then substituted into equation Eqn. (13) to get: $\mathbf{M}\Theta = \mathbf{K}$, then $\Theta = \mathbf{M}^{-1}\mathbf{K}$, where,

$$\mathbf{M} = B_3\bar{\mathbf{D}}^2 + \frac{EcPr}{B_1}\left(\bar{\mathbf{D}}^2\mathbf{F}\right)^2 + \left(A^*\bar{\mathbf{D}}\mathbf{F} + B^*\mathbf{I}\right) - B_4 Pr\left\{\frac{S}{2}\left(\left(\eta\bar{\mathbf{D}} + 3\mathbf{I}\right) + 2\mathbf{I}\bar{\mathbf{D}}\mathbf{F} - \mathbf{F}\bar{\mathbf{D}}\right)\right\} \quad (14)$$

K = [0 0 … 0 0] ^{T}, and **I** is the identity matrix of size $N+1$. The boundary conditions are imposed by setting the first and last row of **M** to be [1 0 … 0 0] and [0 0 …0 1].

3.1 Algorithm for the implementation of the Chebyshev pseudospectral collocation method.

Using the following routine, the Chebyshev pseudospectral collocation method can be implemented as follows:

Step 1: Input the number of collocations points and compute the coordinate values of the nodes for the matrices of the first-order, second-order and third-order derivatives.

Step 2: Set the dimensionless velocity and temperature to the initial values in all directions except at the boundaries.

Step 3: Use Eqns. (12) and (13), assemble the matrices.

Step 4: Imposed the boundary conditions in the algebraic systems of equations and solved the equation directly.

Step 5: Using the convergence criteria, terminate the iteration procedure, otherwise, return to Step 3. The convergence criteria are as follows.

$$\sum\left|\phi_i^p - \phi^{p-1}\right| \le 10^{-12}$$

4. Results and Discussion

The results of the approximate analytical methods of solution for the non-linear thermal model as developed in this work are verified with the results in our previous studies on the work [79]. It was established that there is high accuracy and very good agreement of the results of the present study with our previous work [79]. The numerical solutions are computed to obtain the results accurately. The convergence criteria of the results as given in the previous section is achieved with the desired degree of accuracy. The results of the discussion are illustrated through figures 2 to 11 to substantiate the applicability of the present analysis.

Figure 2 presents the impact of thermal radiation parameter on the thermal profile. It is displayed that an increase in radiation parameter causes the velocity of the fluid to increase, while the temperature profiles increase with increasing radiation parameter values. This is because increases in thermal radiation causes the thermal boundary layer of fluid to increase. Generally, increasing radiation parameter values enhances the temperature near the boundary.

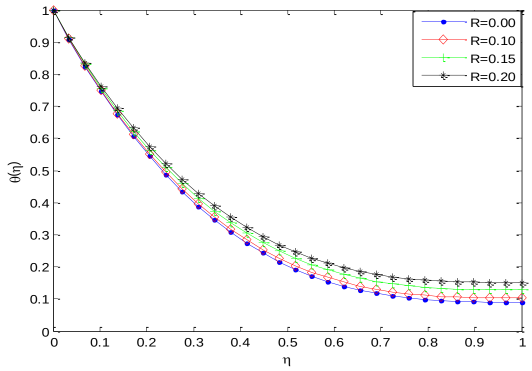

Fig. 2 – Effects of radiation parameter

The significance of Casson parameter on the hydrodynamic profile Casson nanofluid are respectively shown in 3. It is illustrated that the magnitude of velocity near the plate for Casson nanofluid parameter reduces for amplifying value of the Casson parameter, while temperature enlarges for increase in the value of the Casson fluid parameter.

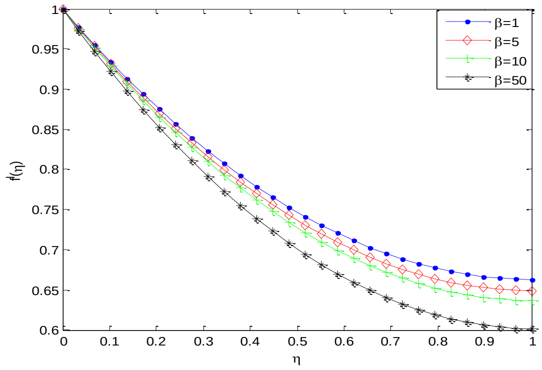

Fig. 3 – Effects of Casson parameter

Figures 4 and 5 show the importance of magnetic field/Hartmann numbers on the hydrodynamic and thermal profiles, respectively. It is illustrated that as the *Ha* number increases, the velocity profile dominates, and this enhances the temperature field. The presence of magnetic field slows fluid motion at boundary layer and hence retards the velocity field. It should be noted that the magnetic field tends to make the boundary layer thinner, thereby increasing the wall friction. It is seen through Fig. 5 that the temperature profile $\theta\ (\eta)$ enhances increasing the Hartmann number Ha.

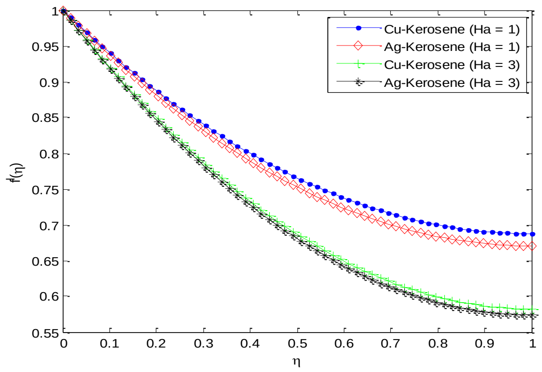

Fig. 4 – Effect of Magnetic field parameter (*Ha*. Number)

Figures 6 and 7 present the influences of the unsteadiness parameter on hydrodynamic and thermal profiles, respectively. It is observed that increasing values of S increases the velocity field while decreases the temperature field. This is because the rate of heat loss by the thin film increases as the value of unsteadiness parameter increases.

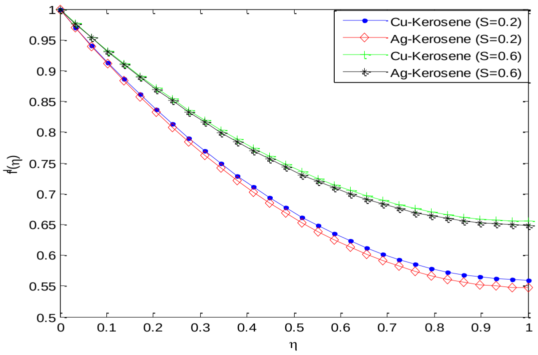

Fig. 6 – Effect of unsteadiness parameter on the fluid velocity distribution

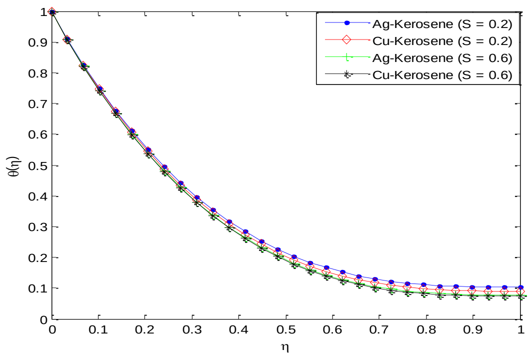

Fig. 7 – Effect of unsteadiness parameter on the fluid temperature distribution

The effects of Weissenberg number (*We*) on the velocity and temperature profiles are shown in figures 8 and 9. It is shown from the figures that the velocity increases for increasing values of *We* and opposite trend was observed in temperature field. The observed trends in the velocity and temperature fields are since a higher value of *We* will reduce the viscosity forces of the Carreau fluid. Increasing the Weissenberg number reduces the magnitude of the fluid velocity for shear thinning fluid while it arises for the shear thickening fluid.

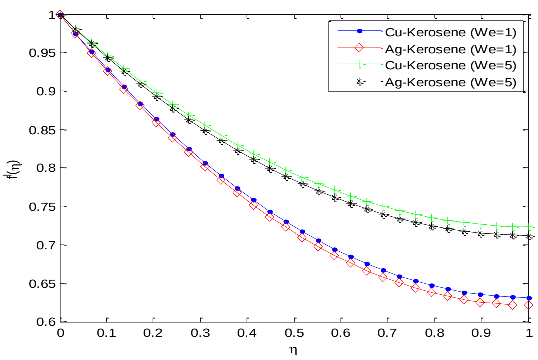

Fig. 8 – Effect of Weissenberg number on the fluid velocity distribution

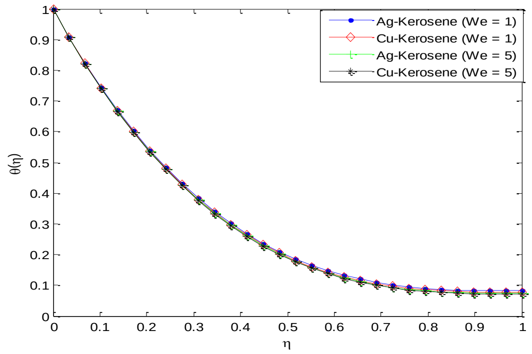

Fig. 9 – Effect of Weissenberg number on the fluid temperature distribution

The influence of aligned angle on velocity and temperature profiles is presented in figures 10 and 11. From the figures, it is shown that as the value of aligned parameter increases, the velocity field increases while temperature field decreases.

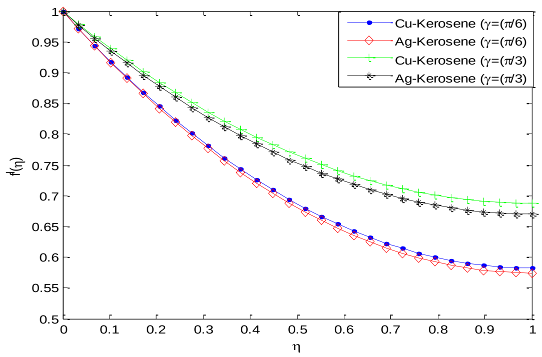

Fig. 10 – Effect of aligned angle on the fluid velocity distribution

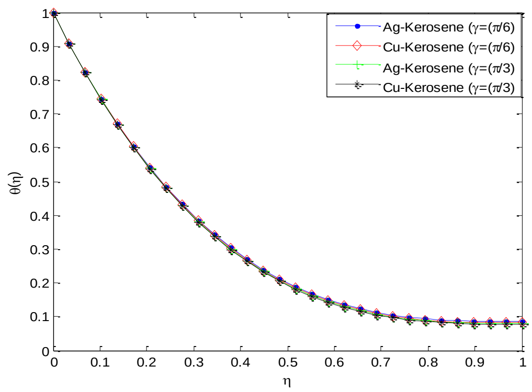

Fig. 11 – Effect of aligned angle on the fluid temperature distribution

5. Conclusion

In this paper, with the aid of Chebyshev pseudospectral collocation method, combined influences of thermal radiation, inclined magnetic field and temperature-dependent internal heat generation on unsteady two-dimensional flow and heat transfer analysis of dissipative Casson-Carreau nanofluid over a stretching sheet embedded in a porous medium have been investigated examined.

Using kerosene as the base fluid embedded with the silver (Ag) and copper (Cu) nanoparticles, the effects of other pertinent parameters on flow and heat transfer characteristics of the nanofluids are investigated and discussed. From the results, it was established temperature field and the thermal boundary layers of Ag-kerosene nanofluid are highly effective when compared with the Cu-kerosene nanofluid. Thermal and momentum boundary layers of Cu-kerosene and Ag-kerosene nanofluids are not uniform. Heat transfer rate is enhanced by increasing in power-law index and unsteadiness parameter. Skin friction coefficient and local Nusselt number can be reduced by magnetic field parameter, and they can be enhanced by increasing in aligned angle.

The friction factor is depreciated and the rate of heat transfer increases by increasing the Weissenberg number. The analysis of the stretched flows with heat transfer is very significant in controlling the qualities of the end products in manufacturing and metal forming processes. Such processes have great dependences on the stretching and cooling rates. The results of present analysis can help in expanding the understanding of the thermo-fluidic behaviors of the Casson-Carreau nanofluid over a stretching sheet as applied in a few industrial manufacturing processes.

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Nomenclature

$\dot{a}$	Suction/injection parameter time-dependent rate
A^*	non-uniform heat generation/absorption parameter
B^*	non-uniform heat generation/absorption parameter
B_O	Electromagnetic induction
Ec	Eckert number
M	Hartmann number or magnetic field parameter
n	Power law index
Pr	Prandtl's number
$\bar{p}$	Pressure
R	Radiation number
Re	Permeation Reynolds number
S	Unsteadiness parameter
t	Dynamic viscosity of the nanofluid, Ns/m^2
$\bar{u}$	Velocity component in x-direction
$\bar{v}$	Velocity component in y-direction
U_w	Fluid inflow velocity at the wall
We	Weissenberg number
$\bar{x}$	Coordinate axis parallel to the channel walls
$\bar{y}$	Coordinate axis perpendicular to the channel walls

Symbol

ρ_{nf}	density of the nanofluid
ρ_f	density of the fluid
μ_{nf}	dynamic viscosity of the nanofluid
ρ_s	density of the nanoparticles
ϕ	fraction of nanoparticles in the nanofluid
σ	electrical conductivity
Γ	time constant
γ	aligned angle

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