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Finite Element Analysis of Transient Thermal Behavior of Radiative-Convective Fin with Functionally Graded Material under the Influence of Magnetic Field

M. G. Sobamowo[1,*], M. O. Salami[1], A. O. Yinusa[1], E. H. Abubakar[2]

[1] Department of Mechanical Engineering,
University of Lagos, Akoka, Lagos State, NIGERIA
[2] Department of Civil Engineering,
University of Lagos, Akoka, Lagos State, NIGERIA

*Corresponding Author

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Abstract: In this work, finite element analysis of transient thermal response of a convective-radiative fin with functionally graded material under the influence of temperature-variant magnetic field is presented. The thermal conductivity of the fin with functional graded fin is linear, quadratic and exponential variation of space. The comparisons of exact analytical solution with the numerical solution show good agreement between the two results. The parametric studies reveal that the temperature of the extended surface increases as the in-homogeneity index. The impact of the in-homogeneity index reduces as the value of the thermo-geometric parameter increases. The change in the fin temperature becomes very insignificant for higher values of the thermo-geometric parameter. This same trend is also noticed under increasing values of the magnetic field and radiative parameters. The augmentations of the radiation and magnetic field parameters leads to decrease in the thermal distribution of the extended surface which in consequent increases the rate of heat transfer through the passive device. The thermal distribution increases as time progresses and later converge to a steady state heat as time evolves. The material under exponential law function needs smaller time to reach steady state than when the material operates under the linear and quadratic law. The results depict that the application of the passive device with functional graded material reduces the thermal resistance along the length of the fin. Therefore, there is a significant higher rate of energy saving in an extended surface with functional graded material than the heat sink with homogenous material.

Keywords: CFD Aerodynamic, CFD Modelling, Numerical methods, small scale turbine

1. Introduction

Extended surfaces are commonly used in practice to enhance heat transfer, and they often increase the rate of heat transfer from a surface several fold. Adding a fin to a system, device or component increases the surface area of the object exposed to the surroundings and hence facilitates the rate of heat transfer. Application of fins can be found in various industrial systems such as the cooling of computer processors, air conditioning and oil carrying pipelines. In recent years, there has been extensive research on moving continuous surface. The continuous and ever-expanding areas in applications of the extended surfaces have instigated a lot of research works [1-8]. An excursion into the previous published studies confirmed that the controlling models for investigating the thermal performance of fins are nonlinear. Consequently, various methods in mathematical analysis have found applications in obtaining solutions to nonlinear models [9-18].

However, the developments of symbolic solutions to the problems often lead to the applications of series solution techniques. Unfortunately, these techniques require high mathematical skills, and the eventual solutions give series solutions with huge expressions and large number of terms [19] which are not appealing to learners and readers. Also, applying such series solution techniques to transient nonlinear problems make the series solutions to be so huge for convenient especially for practical applications. Moreover, such analysis comes with intractable mathematical models involving long time, and great computational cost.

**Corresponding author: mikegbeminiyiprof@yahoo.com*

Furthermore, most of the series solutions methods were applied to heat transfer in steady state for extended surfaces. In most of the previous work, the analytical methods applied to the transient heat transfer problems are limited to linear problems while numerical tools are frequently applied in solving problems in nonlinear transient or unsteady heat transfer. Also, an n in-depth examination and review of the past works indicate to the best of our knowledge that FGM has not been well applied in heat sinks. Also, different studies on the thermal performance of fins with functionally graded materials have been carried out [20-35]. However, to the best of the authors' knowledge, a study on finite element analysis of transient thermal analysis of fin with functionally graded materials has not been considered in literature.

Therefore, finite element analysis of transient thermal analysis of a convective-radiative fin with functionally graded materials under the influence of Lorentz force is presented. The thermal conductivity of the fin with functional graded fin is linear, quadratic, and exponential variation of space. The results are presented and discussed.

2. Problem Formulation

Given a solid rectangular fin that is subjected to thermal convective and radiation effects as shown in figure 1, the fin is acted upon by identical magnetic field placed perpendicularly (*y*-direction) to fin surface. The fin material is taken to be homogenous and isotropic with constant physical and thermal properties except the thermal conductivity.

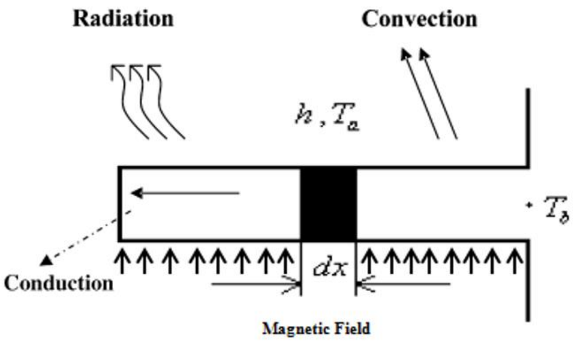

Fig. 1 – Schematic diagram of the magnetic field effects convective-radiation longitudinal rectangular fin

Given the model assumption, the one-directional energy equation in thermal system is as follows.

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\frac{\partial T}{\partial x}\right) - \frac{hP}{A}(T - T_a) - \frac{\sigma\varepsilon P}{A}\left(T^4 - T_a^4\right) - \frac{J_c \times J_c}{\sigma A} \tag{1}$$

But the Lorentz force can be written as,

$$\frac{J_c \times J_c}{\sigma} = \sigma B_o^2 u^2 \tag{2}$$

After substitution of Eqn. 2 into Eqn. 1, taking the magnetic term as a linear function of temperature, we have,

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\frac{\partial T}{\partial x}\right) - \frac{hP}{A}(T - T_a) - \frac{\sigma\varepsilon P}{A}\left(T^4 - T_a^4\right) - \frac{\sigma B_o^2 u^2}{A}(T - T_a) \tag{3}$$

Initial condition is stated as

$$t = 0, \quad \mathrm{T} = T_a, \quad x > 0$$

while the boundary conditions are

$$t > 0, \quad x = 0, \quad \frac{\partial T}{\partial x} = 0$$

$$t > 0, \quad x = b, \quad T = T_b$$

In the present analysis, cases of linear, quadratic, and exponential variation of thermal conductivity with the fin length will be considered. Linear variation of thermal conductivity with the fin length

$$k(x) = k_o\left(1 + \gamma_l x\right)$$

Quadratic variation of thermal conductivity with the fin length,

$$k(x) = k_o\left(1 + \gamma_q x^2\right)$$

Exponential variation of thermal conductivity with the fin length,

$$k(x) = k_o e^{\gamma_i x}$$

Therefore, the thermal models for the linear, quadratic and exponential-law are,

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_o\left(1 + \gamma_l x\right)\frac{\partial T}{\partial x}\right) - \frac{hP}{A}(T - T_a) - \frac{\sigma\varepsilon P}{A}\left(T^4 - T_a^4\right) - \frac{\sigma B_o^2 u^2}{A}(T - T_a) \tag{4}$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_o\left(1 + \gamma_q x^2\right)\frac{\partial T}{\partial x}\right) - \frac{hP}{A}(T - T_a) - \frac{\sigma\varepsilon P}{A}\left(T^4 - T_a^4\right) - \frac{\sigma B_o^2 u^2}{A}(T - T_a) \tag{5}$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_o e^{\gamma_i x}\frac{\partial T}{\partial x}\right) - \frac{hP}{A}(T - T_a) - \frac{\sigma\varepsilon P}{A}\left(T^4 - T_a^4\right) - \frac{\sigma B_o^2 u^2}{A}(T - T_a) \tag{6}$$

To non-dimensionalize the developed governing equations, the following dimensionless parameters are used in Eqns. 4-6,

$$X = \frac{x}{L}, \; \theta = \frac{T - T_a}{T_b - T_a}, \; \tau = \frac{k_o t}{\rho c_p b^2}, \; \beta = \gamma_l b = \gamma_i b, \; \lambda = \beta_q b^2,$$

$$Nc^2 = \frac{Pbh}{Ak_o}, \; Nr = \frac{4\sigma\varepsilon P T_a^3}{k_o A}, \; Ma^2 = \frac{\sigma B_0^2 u^2}{k_o A} \;\; C_T = \frac{T_a}{T_b - T_a}$$

The dimensionless thermal models for the linear, quadratic and exponential-law are,

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial X}\left(\left(1+\gamma_l X\right)\frac{\partial\theta}{\partial X}\right)-Nc^2\theta-Nr\theta^4 \\ -Nr\left[\left(\theta+C_T\right)^4-C_T^4\right]-Ma\theta \tag{7}$$

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial X}\left(\left(1+\gamma_q X^2\right)\frac{\partial\theta}{\partial X}\right)-Nc^2\theta-Nr\theta^4 \\ -Nr\left[\left(\theta+C_T\right)^4-C_T^4\right]-Ma\theta \tag{8}$$

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial X}\left(e^{\gamma_i X}\frac{\partial\theta}{\partial X}\right)-Nc^2\theta-Nr\theta^4 \\ -Nr\left[\left(\theta+C_T\right)^4-C_T^4\right]-Ma\theta \tag{9}$$

and the dimensionless initial and boundary conditions are,

$$\tau=0,\quad \theta=0,\quad X>0$$

$$\tau>0,\quad X=0,\quad \frac{\partial\theta}{\partial X}=0$$

$$\tau>0,\quad X=1,\quad \theta=1$$

Expansion of the nonlinear radiation term in Eqns. 7-9 and gives,

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial X}\left(\left(1+\gamma_l X\right)\frac{\partial\theta}{\partial X}\right)-Nc^2\theta-Nr\theta^4 \\ -4NrC_T\theta^3-6NrC_T^2\theta^2-4NrC_T^3\theta-Ma\theta \tag{10}$$

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial X}\left(\left(1+\gamma_q X^2\right)\frac{\partial\theta}{\partial X}\right)-Nc^2\theta-Nr\theta^4 \\ -4NrC_T\theta^3-6NrC_T^2\theta^2-4NrC_T^3\theta-Ma\theta \tag{11}$$

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial X}\left(e^{\gamma_i X}\frac{\partial\theta}{\partial X}\right)-Nc^2\theta-Nr\theta^4 \\ -4NrC_T\theta^3-6NrC_T^2\theta^2-4NrC_T^3\theta-Ma\theta \tag{12}$$

Generally, we can write Eqns. (10), (11) and (12) as,

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial X}\left(e^{\gamma_i X}\frac{\partial\theta}{\partial X}\right)-Nc^2\theta-Nr\theta^4-4NrC_T\theta^3 \\ -6NrC_T^2\theta^2-4NrC_T^3\theta-Ma\theta \tag{13}$$

3. Finite Element Method to Thermal Model

The strong nonlinearity in Eqn. 13 renders the development of exact analytical solution to the nonlinear equation very difficult. Therefore, Galerkin finite element method is used in this work to solve the nonlinear equation. The procedures of the finite element analysis are summarized as follows:

- discretization of the domain into elements;
- derivation of element equations;
- assembly of element equations;
- imposition of boundary conditions; and
- solution of assembled equations.

Following the procedures of finite element method, the entire computational domain of each profile is partitioned into "*r*" number of linear elements of equivalent size. Then variational formulation of the given problem over the typical element is constructed as given by Eqn. 13.

$$\int_{X_i}^{X_j} w_v \left\{ \begin{array}{c} \frac{\partial}{\partial X}\left(K(X)\frac{\partial\theta}{\partial X}\right)-Nc^2\theta-Nr\theta^4-4NrC_T\theta^3 \\ -6NrC_T^2\theta^2-4NrC_T^3\theta-Ma\theta-\frac{\partial\theta}{\partial\tau} \end{array} \right\} dX=0$$

where w_v is the weight function corresponding to $\theta(x)$. Using the Galerkin finite element method, the shape function is equal to the weight function, Therefore, the above expression can be written as,

$$\int_{X_i}^{X_j} N_v \left\{ \begin{array}{c} \frac{\partial}{\partial X}\left(K(X)\frac{\partial\theta}{\partial X}\right)-Nc^2\theta-Nr\theta^4-4NrC_T\theta^3 \\ -6NrC_T^2\theta^2-4NrC_T^3\theta-Ma\theta-\frac{\partial\theta}{\partial\tau} \end{array} \right\} dX=0$$

where v=i, j.

An approximate solution of the variational problem is assumed, and the element equations are generated by substituting the assumed solution in the formulation as shown from the rest of equations. The element matrix, which is called stiffness matrix, is constructed by using the element interpolation functions as stated as follows:

The finite element approximate solution for a two-node linear element in figure 2 is given as,

$$\theta(X,\tau)=N_i(X)\theta_i(\tau)+N_j(X)\theta_j(\tau) \tag{14}$$

After derivation from the two-node linear element, we have,

$$\theta(X,\tau)=\left(\frac{X_j-X}{X_j-X_i}\right)\theta_i(\tau)+\left(\frac{X-X_i}{X_j-X_i}\right)\theta_j(\tau), \tag{15}$$

where,

$$N_i=\frac{X_j-X}{X_j-X_i},\quad N_j=\frac{X-X_i}{X_j-X_i},$$

N_i and N_j are called shape/interpolation/test/basis functions.

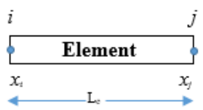

Fig. 2 – Two-nodes element.

For a two-node element as shown in figure 2, with

$$X_i=0,\quad X_j=L_e,\quad \Rightarrow X_j-X_i=L_e,$$

Therefore, the shape functions can be written as,

$$N_i=1-\frac{X}{L_e},\quad N_j=\frac{X}{L_e},$$

We can therefore write Eqn. 15 as,

$$\theta(X,\tau)=\left(1-\frac{X}{L_e}\right)\theta_i(\tau)+\left(\frac{X}{L_e}\right)\theta_j(\tau), \tag{16}$$

The above equation can be written in the finite element model form as,

$$\left[C^e\right]\left\{\frac{d\theta^e}{d\tau}\right\}+\left[K^e\right]\left\{\theta^e\right\}=\left[f^e\right] \tag{17}$$

4. Finite Element Method for Time Discretization

The differential operator contains the time-dependent term, and it must be discretized. Therefore, the transient term is discretized using finite element method as follows. The transient temperature equation is now discretized in the time domain as in figure 3.

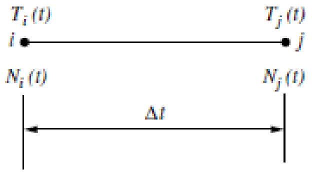

Fig. 3 – Time discretization between n^{th} (*i*) and n + 1th (*j*) time levels

$$\begin{aligned}T(t)&=N_i(t)T_i(t)+N_j(t)T_j(t)\\&=\begin{bmatrix}N_i(t) & N_j(t)\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix},\end{aligned} \tag{18}$$

The linear shape functions are developed as,

$$N_i(t)=1-\frac{t}{\Delta t},\; N_j(t)=\frac{t}{\Delta t}, \tag{19}$$

Then, the time derivative of the temperature equation is derived as,

$$\frac{\partial T(t)}{\partial t}=\begin{bmatrix}-\frac{1}{\Delta t} & \frac{1}{\Delta t}\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}. \tag{20}$$

Substituting Eqns. 18-20 into the Eqn. 17 and using weighted residual principle (Galerkin method), we obtain for a time interval of Δt,

$$\int_{\Delta t}\left[\begin{bmatrix}N_i(t)\\N_j(t)\end{bmatrix}\begin{bmatrix}[\mathbf{C}]\begin{bmatrix}-\frac{1}{\Delta t} & \frac{1}{\Delta t}\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}+[\mathbf{K}(T)]\\ \begin{bmatrix}N_i(t)N_j(t)\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}-[\mathbf{f}(T)]\end{bmatrix}\right]dt$$

The above equation can be simplified as,

$$\int_{\Delta t}\begin{bmatrix}[\mathbf{C}]\begin{bmatrix}N_i(t)\\N_j(t)\end{bmatrix}\begin{bmatrix}-\frac{1}{\Delta t} & \frac{1}{\Delta t}\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}+[\mathbf{K}(T)]\\ \begin{bmatrix}N_i(t)\\N_j(t)\end{bmatrix}\begin{bmatrix}N_i(t) & N_j(t)\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}-\begin{bmatrix}N_i(t)\\N_j(t)\end{bmatrix}[\mathbf{f}(T)]\end{bmatrix}dt,$$

Substituting Eqn. 19 into the above expression,

$$\int_{\Delta t}\begin{bmatrix}[\mathbf{C}]\begin{bmatrix}1-\frac{t}{\Delta t}\\ \frac{t}{\Delta t}\end{bmatrix}\begin{bmatrix}-\frac{1}{\Delta t} & \frac{1}{\Delta t}\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}+[\mathbf{K}(T)]\\ \begin{bmatrix}1-\frac{t}{\Delta t}\\ \frac{t}{\Delta t}\end{bmatrix}\begin{bmatrix}1-\frac{t}{\Delta t} & \frac{t}{\Delta t}\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}-\begin{bmatrix}1-\frac{t}{\Delta t}\\ \frac{t}{\Delta t}\end{bmatrix}[\mathbf{f}(T)]\end{bmatrix}dt$$

One arrives at after expanding the equation.

$$\int_{\Delta t}\begin{bmatrix}[\mathbf{C}]\begin{bmatrix}-\left(1-\frac{t}{\Delta t}\right)\left(\frac{1}{\Delta t}\right) & \left(1-\frac{t}{\Delta t}\right)\left(\frac{1}{\Delta t}\right)\\ -\left(\frac{t}{\Delta t}\right)\left(\frac{1}{\Delta t}\right) & \left(\frac{t}{\Delta t}\right)\left(\frac{1}{\Delta t}\right)\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}+[\mathbf{K}(T)]\\ \begin{bmatrix}\left(1-\frac{t}{\Delta t}\right)^2 & \left(1-\frac{t}{\Delta t}\right)\left(\frac{t}{\Delta t}\right)\\ \left(1-\frac{t}{\Delta t}\right)\left(\frac{t}{\Delta t}\right) & \left(\frac{t}{\Delta t}\right)^2\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}-\begin{bmatrix}1-\frac{t}{\Delta t}\\ \frac{t}{\Delta t}\end{bmatrix}[\mathbf{f}(T)]\end{bmatrix}dt$$

Evaluation of the new expression gives the characteristic equation over the time interval Δt as,

$$\frac{1}{2\Delta t}\begin{bmatrix}[\mathbf{C}]\begin{bmatrix}-1 & 1\\-1 & 1\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}+\frac{1}{3}[\mathbf{K}(T)]\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{Bmatrix}T_i(t)\\T_j(t)\end{Bmatrix}\\ =\frac{1}{2}\begin{bmatrix}1\\1\end{bmatrix}\begin{Bmatrix}f_1\\f_2\end{Bmatrix}\end{bmatrix}$$

The above equation involves the temperature values at the *m th* and (m + 1) *th* level. The matrices are found K_{ij}^e and f_i^e after substitution of $N_i = 1 - X/L_e$ and $N_j = 1 - X/L_e$ into the above equations and then integrate. Thus, the element matrix is obtained. Algebraic equations are developed from the element matrix which are assembled by imposing the inter-element continuity conditions. Thus, a matrix of the order "*r+1'* is developed. Due to the present of nonlinearity term in the finite element model obtained is nonlinear. Therefore, an iterative method is adopted in the solution of the systems of nonlinear equations. The essential and natural boundary conditions given in previous equations are imposed on the assembled equations. When the two boundary conditions are imposed, the remaining system contains "*r-1*" equations, and system that is solved by the Gauss-Seidel method using an accuracy of $10^{-6.}$

5. Results and Discussion

The resulting system of algebraic equations are simulated with MATLAB and parametric studies were explored as shown in figures 4-10. However, before the parametric studies, the numerical results are compared with the results of the exact analytical solution for a linearized model. The comparisons of the results are shown in Table 1 which illustrate excellent agreement between the two sets of the results.

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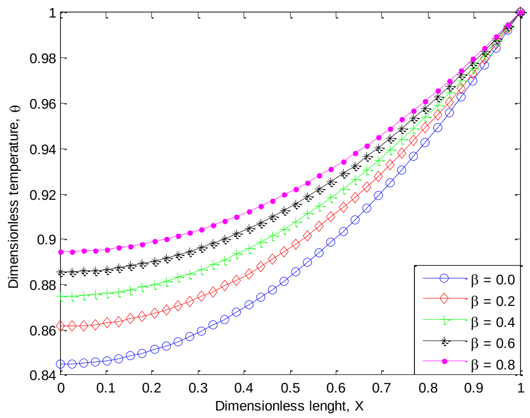
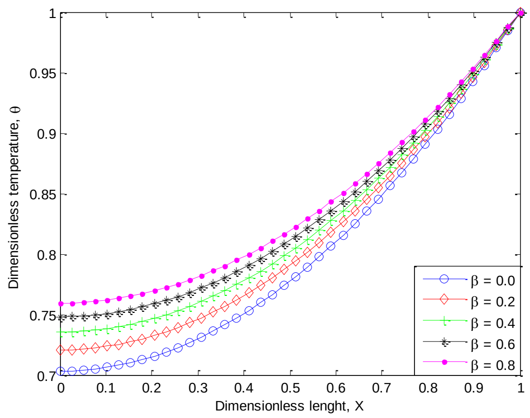
Table 1 – Comparison of the results

X	Exact	FEM
0.00	0.863499158	0.863495683
0.10	0.864817031	0.864817362
0.20	0.868776195	0.868776727
0.30	0.875393336	0.875393986
0.40	0.884696438	0.884696894
0.50	0.896725040	0.896725578
0.60	0.911530606	0.911530958
0.70	0.929177015	0.929177849
0.80	0.949741166	0.949741998
0.90	0.973313722	0.973313941
1.00	1.000000000	1.000000000

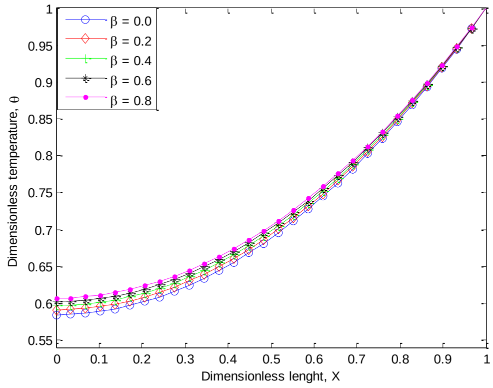
The thermal behavior of the passive device to changes in the in-homogeneity index is depicted in figure 4. It is illustrated that the temperature of the extended surface increases as the in-homogeneity index increases as shown in figure 4(a) and 4(b). It is also shown that the impact of the in-homogeneity index reduces as the value of the thermo-geometric parameter, N_c increases as depicted in figures 4(a) to 4(d). In fact, the change in the fin temperature becomes very insignificant for higher values of the thermo-geometric parameter as shown in figure 4(d). This same trend is also noticed under increasing values of the magnetic field and radiative parameters.



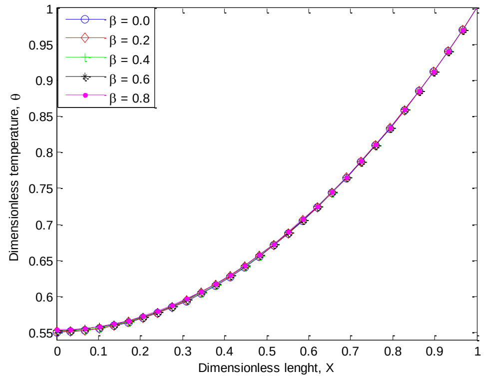
(a)



(b)



(c)



(d)

Fig. 4 – In-homogeneity index impact on the thermal response in the extended surface (a) N_c=1 (b) N_c =2 (c) N_c= 5 (d) N_c=15

The significances magnetic field and radiation parameters on the thermal behavior of the fin are presented in figures 5 and 6. The augmentations of the radiation and magnetic field parameters leads to decrease in the thermal distribution of the extended surface which in consequent increases the rate of heat transfer through the passive device. It is therefore recommended that the fin thermal performance and efficiency can be enhanced by the introduction of magnetic force and thermal radiation in the heat enhancement process.

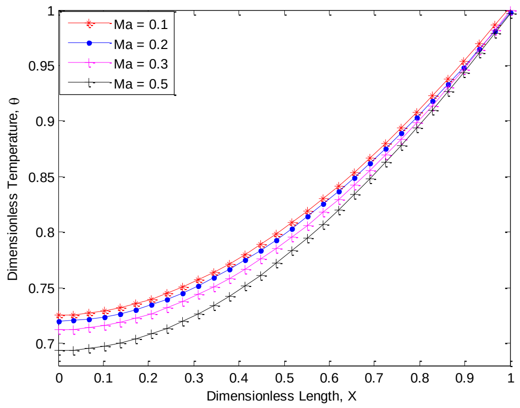


Fig. 5 – Magnetic field parameter influence on the tthermal behaviour

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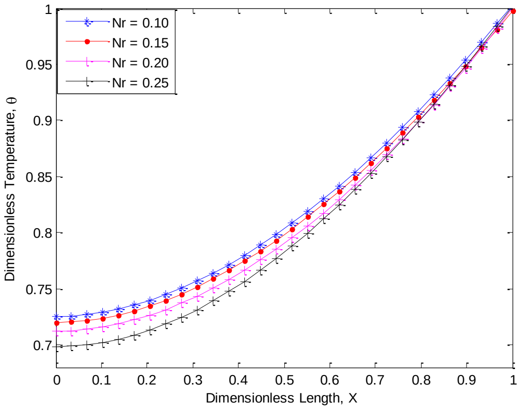


Fig. 6 – Radiation number effect influence on the tthermal behaviour

Figures 7 to 9 analyzed the impacts temperature histories of the heat enhancement device at different positions in the FGM material with variable thermal conductivity according to linear-, quadratic- and exponential-law functions, respectively. As it is expected, the thermal distribution increases as time progresses and later converge to a steady state heat as time evolves. The material under exponential law function needs smaller time to reach steady state than when the material operates under the linear and quadratic law.

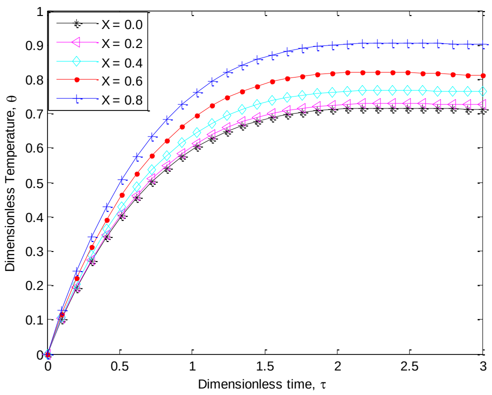


Fig. 7 – Time evolution of the fin temperature history at different locations for the linear-law function

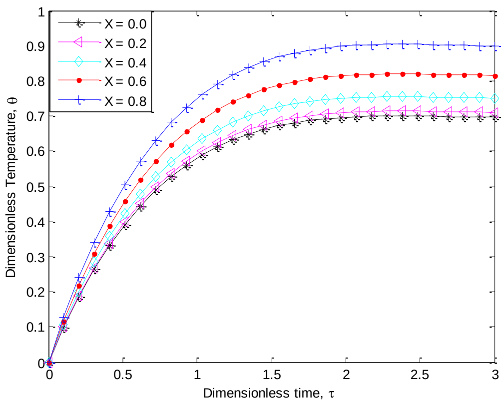


Fig. 8 – Time evolution of the fin temperature history at different locations for the quadratic-law function

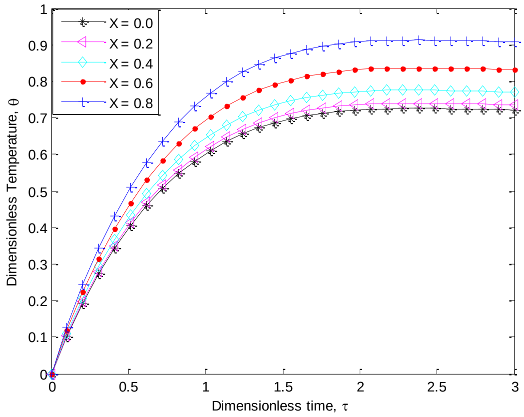
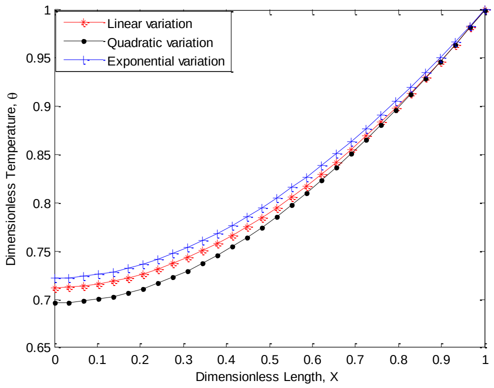
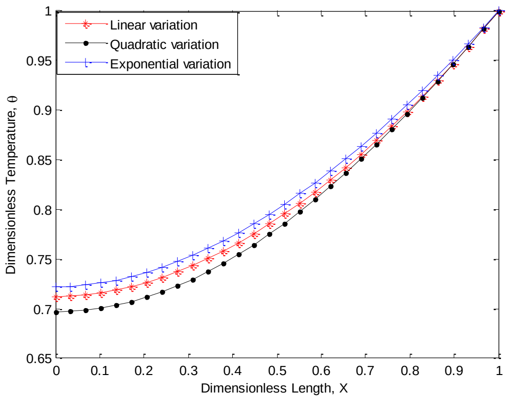


Fig. 9 – Time evolution of the fin temperature history at different locations for the exponential-law function

Figures 10(a) to 10(d), the effects of the thermal conductivity laws on the thermal behavior of the fin at difference dimensionless time are shown. Under quadratic law, the fin behavior shows an enhanced performance of lower temperature distribution as compared to the linear and exponential-law functions. From all the results presented, it is shown that the application of the passive device with FGM reduces the thermal resistance along the length of the fin. Therefore, there is a significant higher rate of energy saving in an extended surface with functional graded material than the heat sink with homogenous material.



(a)



(b)

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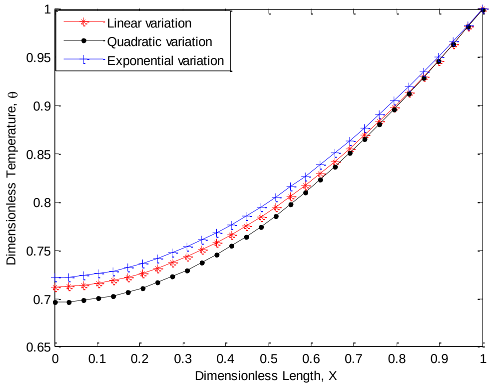
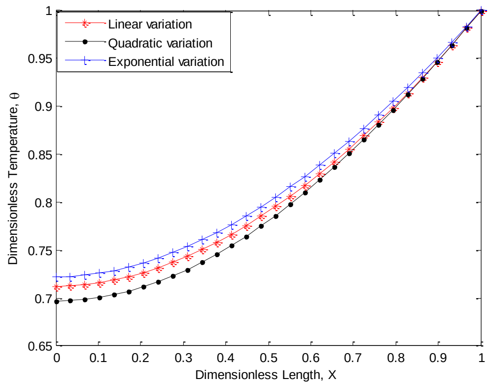


(c)



(d)

Fig. 10 – Fin thermal distribution at different law of thermal conductivity (a) τ = 0.10 (b) τ = 0.15 (c) τ = 0.30 (d) τ = 0.35

6. Conclusion

Numerical analysis of transient thermal behavior of a radiative-convective fin with functionally graded materials under the influence of magnetic field has been presented in this work using finite element analysis. The thermal conductivity of the fin with functional graded fin is linear, quadratic, and exponential variation of space. The parametric studies revealed that;

i. The temperature of the extended surface increases as the in-homogeneity index. The impact of the in-homogeneity index reduces as the value of the thermo-geometric parameter increases.

ii. The change in the fin temperature becomes very insignificant for higher values of the thermo-geometric parameter. This same trend was also recorded under increasing values of the magnetic field and radiative parameters.

iii. The augmentations of the radiation and magnetic field parameters leads to decrease in the thermal distribution of the extended surface which in consequent increases the rate of heat transfer through the passive device.

iv. The thermal distribution increases as time progresses and later converge to a steady state heat as time evolves. The material under exponential law function needs smaller time to reach steady state than when the material operates under the linear and quadratic law.

From the results, it is established that the application of the passive device with functional graded material reduces the thermal resistance along the length of the fin. Therefore, there is a significant higher rate of energy saving in an extended surface with functional graded material than the heat sink with homogenous material.

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